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Factorization and Decay Constants $f_{D_s^*}$ and f_{D_s}

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Abstract

We calculate the decay constants of D_s and D_s^* with $\bar{B}^0 \to D^+\ell^-\nu$ and $\bar{B}^0 \to D^+ D_s^{-(*)}$ decays. In our analysis we take the factorization method with considering non-factorizable term contributions and used two different form factor behaviours (constant and monopole-type) for $F_0(q^2)$. We also consider the QCD-penguin and Electroweak-penguin contributions in hadronic decays within the NDR renormalization scheme at NLO calculation. We estimate the decay constant of the D_s meson to be 233 ± 49 MeV for (pole/pole)-type form factor and 255 ± 54 MeV for (pole/constant)-type form factor. For D_s^* meson, we get $f_{D_s^*} = 346 \pm 82$ MeV, and $f_{D_s^*}/f_{D_s} = 1.43 \pm 0.45$ for (pole/constant)-type form factor.

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1. Introduction

Measuring purely leptonic decays of heavy mesons provides the most clear way for the determination of weak decay constants of heavy mesons, which connect the measured quantities, such as the $B\bar{B}$ mixing ratio, to CKM matrix elements V_{cb}, V_{ub}. However, currently it is not possible to determine f_B, f_{B_s} f_{D_s} and $f_{D_s^*}$ experimentally from leptonic B and D_s decays. For instance, the decay rate for D_s^+ is given by [1]

$$\Gamma(D_s^+ \to \ell^+\nu) = \frac{G_F^2}{8\pi} f_{D_s}^2 m_\ell^2 M_{D_s} \left(1 - \frac{m_l^2}{M_{D_s}^2}\right)^2 |V_{cs}|^2 \qquad (1)$$

Because of helicity suppression, the electron mode $D_s^+ \to e^+\nu$ has a very small rate. The relative widths are $10 : 1 : 2 \times 10^{-5}$ for $\tau^+\nu, \mu^+\nu$ and $e^+\nu$ final states, respectively. Unfortunately the mode with the largest branching fraction, $\tau^+\nu$, has at least two neutrinos in the final state and is difficult to detect in experiment. So theoretical calculations for decay constant have to be used. The factorization ansatz for nonleptonic decay modes provides us a good approximate method to obtain nonperturbative quantities such as form factors and decay constants which are hardly accessible in any other way [2, 3].

There are many ways that the quarks produced in a nonleptonic weak decay can arrange themselves into hadrons. The final state is linked to the initial state by complicated trees of gluon and quark interactions, pair production, and loops. These make the theoretical description of nonleptonic decays difficult. However, since the products of a B meson decay are quite energetic, it is possible that the complicated QCD interactions are less important and that the two quark pairs of the currents in the weak Hamiltonian group individually into the final state mesons without further exchanges of gluons. The color transparency argument suggests that a quark-antiquark pair remains at a state of small size with a correspondingly small chromomagnetic moment until it is far from the other decay products.

Color transparency is the basis for the factorization hypothesis, in which amplitudes factorize into products of two current matrix elements. This ansatz is widely used in heavy quark physics, as it is almost the only way to treat hadronic decays.

In this paper we consider the way how to determine weak decay constants f_{D_s} and $f_{D_s^*}$ under factorization ansatz including penguin effects. In section 2 we discuss the way how to extract the unknown parameter $|V_{cb}F_1^{BD}(0)|$ from the branching ratio of the semileptonic decay $\bar{B}^0 \to D^+\ell\bar{\nu}$. In order to check the validity of the factorization assumption, we study the nonleptonic two body decays, $B \to D\rho, D\pi$ and $DK^{(*)}$ in section 3. In section 4 we calculate f_{D_s} and $f_{D_s^*}$ from $\bar{B}^0 \to D^+D_s^{-(*)}$ decay modes. In our analysis we improve the previous analysis [4] by considering the QCD-penguin and Electroweak-penguin effects of about 13 % for $B \to DD_s$ and 4 % for $B \to DD_s^*$, which are not negligible as discusssed in [5]. Also we follow the gauge independent approach to calculate the effective Wilson coefficients which was studied by pertubative QCD factorization theorem [6].

2. Semileptonic Decay $\bar{B}^0 \to D^+l^-\bar{\nu}$

From Lorentz invariance one finds the decomposition of the hadronic matrix element in terms of hadronic form factors:

$$\begin{aligned} < D^+(p_D)|J_\mu|\bar{B}^0(p_B) > \; &= \; \left[(p_B+p_D)_\mu - \frac{m_B^2-m_D^2}{q^2}q_\mu\right] F_1^{BD}(q^2) \\ &\quad + \frac{m_B^2-m_D^2}{q^2}\, q_\mu \, F_0^{BD}(q^2), \end{aligned} \tag{2}$$

where $J_\mu = \bar{c}\gamma_\mu b$ and $q_\mu = (p_B - p_D)_\mu$. In the rest frame of the decay products, $F_1(q^2)$ and $F_0(q^2)$ correspond to 1^- and 0^+ exchanges, respectively. At $q^2 = 0$ we have the constraint

$$F_1^{BD}(0) = F_0^{BD}(0), \tag{3}$$

since the hadronic matrix element in (2) is nonsingular at this kinematic point.

The q^2 distribution in the semileptonic decay $\bar{B}^0 \to D^+l^-\bar{\nu}$ is written in terms of the hadronic form factor $F_1^{BD}(q^2)$ as

$$\frac{d\Gamma(\bar{B}^0 \to D^+l^-\bar{\nu})}{dq^2} = \frac{G_F^2}{24\pi^3}\,|V_{cb}|^2\,[K(q^2)]^3\,|F_1^{BD}(q^2)|^2, \tag{4}$$

where the q^2 dependent momentum $K(q^2)$ is given by

$$K(q^2) = \frac{1}{2m_B}\,\left[(m_B^2+m_D^2-q^2)^2 - 4m_B^2m_D^2\right]^{1/2}. \tag{5}$$

In the zero lepton mass limit, $0 \le q^2 \le (m_B - m_D)^2$.

For the q^2 dependence of the form factors, Wirbel et al. [7] assumed a simple pole formula for both $F_1(q^2)$ and $F_0(q^2)$ (pole/pole):

$$F_1(q^2) = F_1(0)\,/(1 - \frac{q^2}{m_{F_1}^2}), \qquad F_0(q^2) = F_0(0)\,/(1 - \frac{q^2}{m_{F_0}^2}), \tag{6}$$

with the pole masses

$$m_{F_1} = 6.34 \text{ GeV}, \qquad m_{F_0} = 6.80 \text{ GeV}. \tag{7}$$

Korner and Schuler [8] also adopted the same q^2 dependence of $F_1(q^2)$ and $F_0(q^2)$ given by (6) and (7). On the other hand, the heavy quark effective theory (HQET) gives in the $m_{b,c} \to \infty$ limit the relation between $F_1(q^2)$ and $F_0(q^2)$ given by [9, 10]

$$F_0(q^2) = \left[1 - \frac{q^2}{(m_B + m_D)^2}\right] \; F_1(q^2). \tag{8}$$

The combination of (6) and (8) suggests that $F_0(q^2)$ is approximately constant when we keep the simple pole dependence for $F_1(q^2)$. Therefore, in this paper, as well as the above (pole/pole) form factors, we will also consider the following ones (pole/const.):

$$F_1(q^2) = F_1(0)\,/(1 - \frac{q^2}{m_{F_1}^2}), \qquad F_0(q^2) = F_0(0), \tag{9}$$

with

$$m_{F_1} = 6.34 \text{ GeV}. \tag{10}$$

By introducing the variable $x \equiv q^2/m_B^2$, which has the range of $0 \le x \le (1 - \frac{m_D}{m_B})^2$ in the zero lepton mass limit, (4) is written as

$$\begin{aligned} \frac{d\Gamma(\bar{B}^0 \to D^+ l^- \bar{\nu})}{dx} &= \frac{G_F^2 m_B^5}{192\pi^3}\, |V_{cb}\, F_1^{BD}(0)|^2\, \frac{\lambda^3[1, \frac{m_D^2}{m_B^2}, x]}{\left(1 - \frac{m_B^2}{m_{F_1}^2} x\right)^2}, \\ \lambda[1, \frac{m_D^2}{m_B^2}, x] &= \left[(1 + \frac{m_D^2}{m_B^2} - x)^2 - 4\frac{m_D^2}{m_B^2}\right]^{1/2}. \end{aligned} \tag{11}$$

Then the branching ratio $\mathcal{B}(\bar{B}^0 \to D^+ l^- \bar{\nu})$ is given by

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \to D^+ l^- \bar{\nu}) &= (\frac{G_F m_B^2}{\sqrt{2}})^2\, \frac{m_B}{\Gamma_B}\, \frac{2}{192\pi^2}\, |V_{cb}\, F_1^{BD}(0)|^2 \times I \\ &= 2.221 \times 10^2\, |V_{cb}\, F_1^{BD}(0)|^2 \times I, \end{aligned} \tag{12}$$

where the dimensionless integral I is given by

$$I = \int_0^{(1-\frac{m_D}{m_B})^2} dx \frac{\left[(1+\frac{m_D^2}{m_B^2}-x)^2 - 4\frac{m_D^2}{m_B^2}\right]^{3/2}}{\left(1-\frac{m_B^2}{m_{F_1}^2}x\right)^2} = 0.121 \tag{13}$$

In obtaining the numerical values in (12) and (13), we used the following experimental results [11]: $m_D = m_{D^+} = 1.869$ GeV, $m_B = m_{B^0} = 5.279$ GeV, $\Gamma_B = \Gamma_{B^0} = 4.219 \times 10^{-13}$ GeV ($\tau_{B^0} = (1.56 \pm 0.06) \times 10^{-12}$ s), and $G_F = 1.166\ 39(2) \times 10^{-5}$ GeV^{-2}. Since $\mathcal{B}(\bar{B}^0 \to D^+ l^- \bar{\nu}) = (1.78 \pm 0.20 \pm 0.24) \times 10^{-2}$ was obtained experimentally, the value of $|V_{cb} F_1^{BD}(0)|$ can be extracted from (12). Following this procedure, we obtain [12]

$$|V_{cb} F_1^{BD}(0)| = (2.57 \pm 0.14 \pm 0.17) \times 10^{-2}. \tag{14}$$

In the calculations of the next sections, we will use $|V_{cb} F_1^{BD}(0)| = (2.57 \pm 0.22) \times 10^{-2}$ which is given by combining the statistical and systematic errors in (14).

3. Test of Factorization with $\bar{B}^0 \to D^+\rho^-$ and $\bar{B}^0 \to D^+\pi^-$, and Prediction of Branching Ratio $\mathcal{B}(\bar{B}^0 \to D^+ K^{-(*)})$

In general the test of factorization, independent of the numerical values of a_1, a_2 and of the CKM parameters $|V_{cb}|$ or $|V_{ub}|$, can be carried out by considering the ratios of rates for two Class I or Class II B-meson hadronic two-body decays. On the other hand, we can also use the relation between the semi-leptonic decays and the non-leptonic decays with a_1 and a_2 given by other sources. In our analysis we use the latter one.

Let us start by recalling the relevant effective weak Hamiltonian:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [C_1(\mu)\mathcal{O}_1 + C_2(\mu)\mathcal{O}_2] \ + \ \text{H.C.}, \tag{15}$$

$$\mathcal{O}_1 = (\bar{d}\Gamma^\rho u)(\bar{c}\Gamma_\rho b), \quad \mathcal{O}_2 = (\bar{c}\Gamma^\rho u)(\bar{d}\Gamma_\rho b), \tag{16}$$

where G_F is the Fermi coupling constant, V_{cb} and V_{ud} are corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and $\Gamma_\rho = \gamma_\rho(1-\gamma_5)$. The Wilson coefficients $C_1(\mu)$ and $C_2(\mu)$

incorporate the short-distance effects arising from the renormalization of $\mathcal{H}_{\rm eff}$ from $\mu = m_W$ to $\mu = O(m_b)$. By using the Fierz transformation under which $V-A$ currents remain $V-A$ currents, we get the following equivalent forms:

$$\begin{aligned} C_1\mathcal{O}_1 + C_2\mathcal{O}_2 &= (C_1 + \frac{1}{N_c}C_2)\mathcal{O}_1 + C_2(\bar{d}\Gamma^\rho T^a u)(\bar{c}\Gamma_\rho T^a b) \\ &= (C_2 + \frac{1}{N_c}C_1)\mathcal{O}_2 + C_1(\bar{c}\Gamma^\rho T^a u)(\bar{d}\Gamma_\rho T^a b), \end{aligned} \tag{17}$$

where $N_c = 3$ is the number of colors and T^a's are $SU(3)$ color generators. The second terms in (17) involve color-octet currents. In the factorization assumption, these terms are neglected and $\mathcal{H}_{\rm eff}$ is rewritten in terms of "factorized hadron operators" [7]:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*\Big(a_1[\bar{d}\Gamma^\rho u]_H[\bar{c}\Gamma_\rho b]_H + a_2[\bar{c}\Gamma^\rho u]_H[\bar{d}\Gamma_\rho b]_H\Big) \ + \ \text{H.C.}, \tag{18}$$

where the subscript H stands for *hadronic* implying that the Dirac bilinears inside the brackets be treated as interpolating fields for the mesons and no further Fierz-reordering need be done. The phenomenological parameters a_1 and a_2 are related to C_1 and C_2 by

$$a_1 = C_1 + \frac{1}{N_c}C_2, \quad a_2 = C_2 + \frac{1}{N_c}C_1. \tag{19}$$

From the analyses of A.J. Buras [13], the parameters a_1 and a_2 are determined at NLO calculation in the NDR scheme as

$$a_1 = 1.02 \pm 0.01, \quad a_2 = 0.20 \pm 0.05. \tag{20}$$

For the two body decay, in the rest frame of initial meson the differential decay rate is given by

$$d\Gamma = \frac{1}{32\pi^2}|\mathcal{M}|^2\frac{|\mathbf{p}_1|}{M^2}d\Omega, \tag{21}$$

$$|\mathbf{p}_1| = \frac{[(M^2-(m_1+m_2)^2)(M^2-(m_1-m_2)^2)]^{1/2}}{2M}, \tag{22}$$

where M is the mass of initial meson, and m_1 (m_2) and $\mathbf{p}_1$ are the mass and momentum of one of final mesons. By using (2), (18) and $< 0|\Gamma_\mu|\rho(q,\varepsilon) >= \varepsilon_\mu(q) m_\rho f_\rho$, (21) gives the following formula for the branching ratio of $\bar{B}^0 \to D^+\rho^-$:

$$\mathcal{B}(\bar{B}^0 \to D^+\rho^-) \ = \ \left(\frac{G_F m_B^2}{\sqrt{2}}\right)^2 |V_{ud}|^2 \, \frac{1}{16\pi}\, \frac{m_B}{\Gamma_B}\, a_1^2\, \frac{f_\rho^2}{m_B^2}\, |V_{cb}\, F_1^{BD}(m_\rho^2)|^2$$

$$
\begin{aligned}
&\times \quad \left[\Big(1-(\frac{m_D+m_\rho}{m_B})^2\Big)\Big(1-(\frac{m_D-m_\rho}{m_B})^2\Big)\right]^{3/2} \\
&= \quad 13.25 \times |V_{cb}\, F_1^{BD}(m_\rho^2)|^2 \times \left(\frac{a_1}{1.02}\right)^2 . \qquad (23)
\end{aligned}
$$

In obtaining the numerical values in (23), we used the experimental results given below (13), $m_\rho = m_{\rho^+} = 766.9$ MeV, $f_\rho = f_{\rho^+} = 216$ MeV, and $V_{ud} = 0.9751$ [11]. For the value of a_1 we used the value given in (20). Then, by using the formula (23) with the values of $|V_{cb}\, F_0^{BD}(0)|^2$ $(F_0^{BD}(0) = F_1^{BD}(0))$ given in (14), we obtain the branching ratio $\mathcal{B}(\bar{B}^0 \to D^+\rho^-)$ presented in Table 1.

For the process $\bar{B}^0 \to D^+K^{*-}$, by using $< 0|\Gamma_\mu|K^*(q,\varepsilon) >= \varepsilon_\mu(q) m_{K^*} f_{K^*}$, we have

$$
\begin{aligned}
\mathcal{B}(\bar{B}^0 \to D^+K^{*-}) \quad &= \quad \left(\frac{G_F m_B^2}{\sqrt{2}}\right)^2 |V_{us}|^2 \,\frac{1}{16\pi}\,\frac{m_B}{\Gamma_B}\, a_1^2\, \frac{f_{K^*}^2}{m_B^2}\, |V_{cb}\, F_1^{BD}(m_{K^*}^2)|^2 \\
&\times \quad \left[\Big(1-(\frac{m_D+m_{K^*}}{m_B})^2\Big)\Big(1-(\frac{m_D-m_{K^*}}{m_B})^2\Big)\right]^{3/2} \\
&= \quad 0.67 \times |V_{cb}\, F_1^{BD}(m_{K^*}^2)|^2 \times \left(\frac{a_1}{1.02}\right)^2 . \qquad (24)
\end{aligned}
$$

where we used $m_{K^*} = m_{K^{*-}} = 891.59$ MeV, $f_{K^*} = f_{K^{*-}} = 218$ MeV, and $V_{us} = 0.2215$ [11]. By using (24) with $|V_{cb}\, F_1^{BD}(0)|^2$ in (14), we obtain the branching ratio $\mathcal{B}(\bar{B}^0 \to D^+K^{*-})$ presented in Table 1.

By using (2), (18) and $< 0|\Gamma_\mu|\pi(q) >= i q_\mu f_\pi$, (21) gives the following formula for the branching ratio of the process $\bar{B}^0 \to D^+\pi^-$:

$$
\begin{aligned}
\mathcal{B}(\bar{B}^0 \to D^+\pi^-) \quad &= \quad \left(\frac{G_F m_B^2}{\sqrt{2}}\right)^2 |V_{ud}|^2 \,\frac{1}{16\pi}\,\frac{m_B}{\Gamma_B}\, a_1^2\, \frac{f_\pi^2}{m_B^2}\, |V_{cb}\, F_0^{BD}(m_\pi^2)|^2 \\
&\times \Big(1-\frac{m_D^2}{m_B^2}\Big)^2 \left[\Big(1-(\frac{m_D+m_\pi}{m_B})^2\Big)\Big(1-(\frac{m_D-m_\pi}{m_B})^2\Big)\right]^{1/2} \\
&= \quad 5.42 \times |V_{cb}\, F_0^{BD}(m_\pi^2)|^2 \times \left(\frac{a_1}{1.02}\right)^2 , \qquad (25)
\end{aligned}
$$

where we used $m_\pi = m_{\pi^-} = 139.57$ MeV and $f_\pi = f_{\pi^-} = 131.74$ MeV [11]. By using the formula (25) with the values of $|V_{cb}\, F_0^{BD}(0)|^2$ $(F_0^{BD}(0) = F_1^{BD}(0))$ in (14), we obtain the branching ratio $\bar{B}^0 \to D^+\pi^-$ presented in Table 1.

For the process $\bar{B}^0 \to D^+K^-$, by using $< 0|\Gamma_\mu|K^-(q) >= i q_\mu f_{K^-}$, we have

$$
\mathcal{B}(\bar{B}^0 \to D^+K^-) \quad = \quad \left(\frac{G_F m_B^2}{\sqrt{2}}\right)^2 |V_{us}|^2 \,\frac{1}{16\pi}\,\frac{m_B}{\Gamma_B}\, a_1^2\, \frac{f_K^2}{m_B^2}\, |V_{cb}\, F_0^{BD}(m_K^2)|^2
$$

$$\times\Big(1-\frac{m_D^2}{m_B^2}\Big)^2\ \Big[\Big(1-(\frac{m_D+m_K}{m_B})^2\Big)\Big(1-(\frac{m_D-m_K}{m_B})^2\Big)\Big]^{1/2}$$
$$= \ 0.41\times|V_{cb}\,F_0^{BD}(m_K^2)|^2\times\left(\frac{a_1}{1.02}\right)^2. \tag{26}$$

where we used $m_K = m_{K^-} = 493.68$ MeV, $f_K = f_{K^+} = 160.6$MeV [11]. By using (26) with $|V_{cb}\,F_1^{BD}(0)|^2$ in (14), we obtain the branching ratio $\mathcal{B}(\bar{B}^0 \to D^+K^-)$ presented in Table 1. It seems that the factorization method works well in $\bar{B}^0 \to D^+\pi^-, D^+\rho^-$ decays. We predict branching ratios :

$$\mathcal{B}(\bar{B}^0 \to D^+K^-) \simeq 2.7\cdot 10^{-4}\cdot\left(\frac{a_1}{1.02}\right)^2$$
$$\mathcal{B}(\bar{B}^0 \to D^+K^{*-}) \simeq 4.6\cdot 10^{-4}\cdot\left(\frac{a_1}{1.02}\right)^2 \tag{27}$$

which is certainly reachable in near future.

4. Determination of $f_{D_s^*}$ and f_{D_s} from $\bar{B}^0 \to D^+D_s^{-*}$ and $\bar{B}^0 \to D^+D_s^-$

The effective Hamiltonian for $\triangle B = 1$ transitions is given by

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}[V_{ub}V_{uq}^*(C_1O_1^u + C_2O_2^u) + V_{cb}V_{cq}^*(C_1O_1^c + C_2O_2^c) - V_{tb}V_{tq}^*\sum_{i=3}^{6}C_iO_i], \tag{28}$$

where $q = d, s$ and C_i are the Wilson coefficients evaluated at the renormalization scale μ, and the current-current operators $O_1^{u,c}$ and $O_2^{u,c}$ are

$$O_1^u = (\bar{u}_\alpha b_\alpha)_{V-A}(\bar{q}_\beta u_\beta)_{V-A} \qquad O_1^c = (\bar{c}_\alpha b_\alpha)_{V-A}(\bar{q}_\beta c_\beta)_{V-A}$$
$$O_2^u = (\bar{u}_\beta b_\alpha)_{V-A}(\bar{q}_\alpha u_\beta)_{V-A} \qquad O_2^c = (\bar{c}_\beta b_\alpha)_{V-A}(\bar{q}_\alpha c_\beta)_{V-A}, \tag{29}$$

and the QCD penguin operators $O_3 \ - \ O_6$ are

$$O_3 = (\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\beta)_{V-A} \qquad O_4 = (\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\alpha q'_\beta)_{V-A}$$
$$O_5 = (\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\beta)_{V+A} \qquad O_6 = (\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\alpha q'_\beta)_{V+A}. \tag{30}$$

The electroweak penguin operators $O_7 \; - \; O_{10}$ are given by :

$$O_7 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} \frac{3}{2} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A} \qquad O_8 = (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} \frac{3}{2} e_{q'} (\bar{q}'_\alpha q'_\beta)_{V+A}$$
$$O_9 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} \frac{3}{2} e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A} \qquad O_8 = (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} \frac{3}{2} e_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A}. \tag{31}$$

In (28) we consider the effects of the electroweak penguin operators, however, we neglect the contribution of the dipole operators, since its contribution is not important in this work.

When we take $m_t = 174$ GeV, $m_b = 5.0$ GeV, $\alpha_{\rm s}(M_z) = 0.118$ and $\alpha_{\rm em}(M_z) = 1/128$, the numerical values of the renormalization scheme independent Wilson coefficients $\bar{C}_i$ at $\mu = m_b$ are given by [14]

$$\begin{aligned}
&\bar{C}_1 = -0.3125, \quad \bar{C}_2 = 1.1502, \\
&\bar{C}_3 = 0.0174, \quad \bar{C}_4 = -0.0373, \quad \bar{C}_5 = 0.0104, \quad \bar{C}_6 = -0.0459, \\
&\bar{C}_7 = -1.050 \times 10^{-5}, \quad \bar{C}_8 = 3.839 \times 10^{-4}, \\
&\bar{C}_9 = -0.0101, \quad \bar{C}_{10} = 1.959 \times 10^{-3}.
\end{aligned} \tag{32}$$

The effective Hamiltonian in (28) for the decays $\bar{B}^0 \to D^+ D_s^{-(*)}$ can be rewritten as

$$H_{eff} = \frac{G_F}{\sqrt{2}} [V_{cb} V_{cs}^* (C_1^{eff} O_1^c + C_2^{eff} O_2^c) - V_{tb} V_{ts}^* \sum_{i=3}^{10} C_i^{eff} O_i], \tag{33}$$

where C_i^{eff} are given by [15]

$$\begin{aligned}
&C_1^{eff} = \bar{C}_1, \quad C_2^{eff} = \bar{C}_2, \quad C_3^{eff} = \bar{C}_3 - P_{\rm s}/N_{\rm c}, \quad C_4^{eff} = \bar{C}_4 + P_{\rm s}, \\
&C_5^{eff} = \bar{C}_5 - P_{\rm s}/N_{\rm c}, \quad C_6^{eff} = \bar{C}_6 + P_{\rm s}, \quad C_7^{eff} = \bar{C}_7 + P_{\rm e}, \quad C_8^{eff} = \bar{C}_8, \\
&C_9^{eff} = \bar{C}_9 + P_{\rm e}, \quad C_{10}^{eff} = \bar{C}_{10}.
\end{aligned} \tag{34}$$

with

$$\begin{aligned}
&P_{\rm s} = \frac{\alpha_{\rm s}}{8\pi} [\frac{10}{9} - G(m_q, q^2, \mu)] \bar{C}_2(\mu), \\
&P_{\rm e} = \frac{\alpha_{\rm em}}{9\pi} [\frac{10}{9} - G(m_q, q^2, \mu)] (3\bar{C}_1(\mu) + \bar{C}_2(\mu)), \qquad (35) \\
&G(m_q, q^2, \mu) = -4 \int_0^1 x(1-x) \ln(\frac{m_q^2 - x(1-x)q^2}{\mu^2}) dx,
\end{aligned}$$
$$\tag{36}$$

where q denotes the momentum of the virtual gluons appearing in the QCD time-like matrix elements, and N_c is the number of colors. Assuming $q^2 = m_b^2/2$, we obtain the analytic formular for $G(m_q, q^2, \mu)$:

$$G(m_q, \frac{m_b^2}{2}, \mu = m_b) = -\frac{2}{3}\ln\left(\frac{y}{8}\right) + \frac{10}{9} + \frac{2}{3}y + \frac{(2+y)\sqrt{1-y}}{3}\left[\ln\left|\frac{1-\sqrt{1-y}}{1+\sqrt{1-y}}\right| + i\pi\right] \qquad (37)$$

with $y = 8m_q^2/m_b^2$.

By considering the non-factorizable term contributions, the relation between the effective coefficients a_i^{eff} and the Wilson coefficients in the effective Hamiltonian are given by

$$a_{2i}^{eff} = C_{2i}^{eff} + \frac{1}{N_c^{eff}} C_{2i-1}^{eff}, \qquad a_{2i-1}^{eff} = C_{2i-1}^{eff} + \frac{1}{N_c^{eff}} C_{2i}^{eff}. \qquad (38)$$

where $i = 1, ..., 5$, and the non-factorizable effects are absorbed into the N_c^{eff} by

$$\frac{1}{{N_c^{eff}}_i} \equiv \frac{1}{N_c} + \chi_i, \qquad N_c = 3. \qquad (39)$$

In order to simplify the notation, we will use the notation a^i instead of a_i^{eff} in the below.

In usual factorization approach, when we consider the off-shell momentum of the external quark line, the effective Wilson coefficients has the ambiguities of the infrared cutoff and gauge dependence. As stressed by [16], the gauge and infrared dependence always appears as long as the matrix elements of operators are calculated between quark states. Recently this problem was sloved by pertubative QCD factorization theorm [6] by using the on-shell external quark. By following their approach and inserting the values for $m_q = m_c(\mu) = 0.95$ GeV, we get the values $C_i^{eff} (i = 1 \sim 10)$ for $b \to c$ given in Table 2. For different combinations of $N_c^{eff} = 2, 3$, and 5, the values of the effective coefficients $a_i (i = 1 \sim 10)$ are shown in Table 3. Here $(N_c)_{LL,LR} = 3$ corresponds to the naive factorization approximation without considering non-factorizable contributions.

The decay amplitude $\mathcal{A}(\bar{B}^0 \to D^+ D_s^-) \equiv < D^+ D_s^- | \mathcal{H}_{\rm eff} | \bar{B}^0 >$ is given as follows:

$$\begin{aligned} \mathcal{A}(\bar{B}^0 \to D^+ D_s^-) &= \frac{G_F}{\sqrt{2}} [V_{cb} V_{cs}^* a_1 - V_{tb} V_{ts}^* (a_4 + a_{10} + 2(a_6 + a_8) \frac{m_{D_s}^2}{(m_b - m_c)(m_s + m_c)})] \ \mathcal{M}_a \\ &\simeq \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \ a_1 \ R_{DDs} \ \mathcal{M}_a \qquad (40) \end{aligned}$$

where

$$R_{DDs} = \left[1 + \frac{(a_4 + a_{10})}{a_1} + 2\frac{(a_6 + a_8)}{a_1}\frac{m_{D_s}^2}{(m_b - m_c)(m_s + m_c)}\right] \tag{41}$$

and

$$\mathcal{M}_a =< D_s^-|\bar{s}\gamma^\mu\gamma_5 c|0 >< D^+|\bar{c}\gamma_\mu b|\bar{B}^0 >= -if_{D_s}(m_B^2 - m_D^2)F_0^{BD}(m_{D_s}^2) \tag{42}$$

On the other hand, we have

$$\begin{aligned}\mathcal{A}(\bar{B}^0 \to D^+ D_s^{-*}) &= \frac{G_F}{\sqrt{2}}[V_{cb}V_{cs}^* a_1 - V_{tb}V_{ts}^*(a_4 + a_{10})]\ \mathcal{M}_b \\ &\simeq \frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*\ a_1\ R_{DDs^*}\ \mathcal{M}_b \end{aligned} \tag{43}$$

where

$$R_{DDs^*} = \left(1 + \frac{(a_4 + a_{10})}{a_1}\right) \tag{44}$$

and

$$\mathcal{M}_b =< D_s^*|\bar{s}\gamma^\mu\gamma_5 c|0 >< D^+|\bar{c}\gamma_\mu b|\bar{B}^0 >= m_{D_s^*} f_{D_s^*}[\epsilon(q)\cdot(p_B + p_D)]F_1^{BD}(m_{D_s^*}^2). \tag{45}$$

We can estimate the penguin contributions for each process, for exapmle, in the case of $N_{LL} = 2$ and $N_{LR} = 5$:

$$\text{For } \bar{B}^0 \to D^+ D_s^-; \qquad \left|\frac{\mathcal{A}_P}{\mathcal{A}_T}\right| = \left|\frac{(a_4 + a_{10})}{a_1} + 2\frac{(a_6 + a_8)}{a_1}\frac{m_{D_s}^2}{(m_b - m_c)(m_c + m_s)}\right| = 13.1\% \tag{46}$$

$$\text{For } \bar{B}^0 \to D^+ D_s^{*-}; \qquad \left|\frac{\mathcal{A}_P}{\mathcal{A}_T}\right| = \left|\frac{(a_4 + a_{10})}{a_1}\right| = 3.9\% \tag{47}$$

where $\mathcal{A}_T(\mathcal{A}_P)$ stands for the amplitude of tree diagram (penguin diagram). Here we used the values $m_c(m_b) = 0.95$ GeV and $m_s(m_b) = 90$ MeV. Therefore, the penguin contributions affect the extraction of the decay constants f_{D_s} and $f_{D_s^*}$. The penguin contributions for $B \to DD_s$ is more than three times of those for $B \to DD_s^*$.

From (40) and (43) the decay constants are given by

$$\begin{aligned} f_{D_s^*} &= (0.87\times 10^{-1}\ \text{GeV})\cdot\frac{\sqrt{\mathcal{B}(\bar{B}^0 \to D^+ D_s^{-*})}}{|V_{cb}\,F_1^{BD}(m_{D_s^*}^2)|}\cdot\left(\frac{1.02}{a_1}\right)\cdot\frac{1}{R_{DDs^*}}, \\ f_{D_s} &= (0.64\times 10^{-1}\ \text{GeV})\cdot\frac{\sqrt{\mathcal{B}(\bar{B}^0 \to D^+ D_s^{-})}}{|V_{cb}\,F_0^{BD}(m_{D_s}^2)|}\cdot\left(\frac{1.02}{a_1}\right)\cdot\frac{1}{R_{DDs}}. \end{aligned} \tag{48}$$

Browder et al. [17] presented the following experimental results for the branching ratios:

$$\begin{aligned}\mathcal{B}(\bar{B}^0 \to D^+ D_s^{-*}) &= (1.14 \pm 0.42 \pm 0.28) \times 10^{-2} = (1.14 \pm 0.50) \times 10^{-2}, \\ \mathcal{B}(\bar{B}^0 \to D^+ D_s^{-}) &= (0.74 \pm 0.22 \pm 0.18) \times 10^{-2} = (0.74 \pm 0.28) \times 10^{-2}, \end{aligned} \tag{49}$$

where we combined the statistical and systematic errors. From (41),(44),(48), and (49), we obtain the results which are obtained by including the penguin contributions:

$$\begin{aligned} f_{D_s^*} &= 346 \pm 82 \text{ MeV}, \qquad f_{D_s} = 233 \pm 49 \text{ MeV} \qquad \text{for (pole/pole)}, \\ f_{D_s^*} &= 346 \pm 82 \text{ MeV}, \qquad f_{D_s} = 255 \pm 54 \text{ MeV} \qquad \text{for (pole/const.)}. \end{aligned} \tag{50}$$

From (41), (44) and (48) the ratio of the vector and pseudoscalar decay constants $f_{D_s^*}/f_{D_s}$ is given by

$$\frac{f_{D_s^*}}{f_{D_s}} = 1.36 \cdot \frac{|V_{cb} F_0^{BD}(m_{D_s}^2)|}{|V_{cb} F_1^{BD}(m_{D_s^*}^2)|} \cdot \left[\frac{\mathcal{B}(\bar{B}^0 \to D^+ D_s^{-*})}{\mathcal{B}(\bar{B}^0 \to D^+ D_s^{-})}\right]^{1/2} \cdot \left(\frac{0.87}{0.96}\right), \tag{51}$$

which gives

$$\begin{aligned} \frac{f_{D_s^*}}{f_{D_s}} &= 1.56 \pm 0.49 \qquad \text{for (pole/pole)}, \\ \frac{f_{D_s^*}}{f_{D_s}} &= 1.43 \pm 0.45 \qquad \text{for (pole/const.)}. \end{aligned} \tag{52}$$

The decay constant is changed according to the q^2 behaviour of the form factor $F_0(q^2)$. However the amount of change is less than 10% as shown in (50). From this we know that the decay constant is not so much dependent on the behaviour of the form factor. Also when we consider the uncertainty from non-factorizable effects, the decay constant is changed within 10% discrepancy. In table 4 we show the results of $f_{D_s^*}$, f_{D_s} and $f_{D_s^*}/f_{D_s}$ for different non-factorizable contributions.

As discussed in [4], when we consider the penguin contributions with non-factorizable effects, the value of the decay constant $f_{D_s^*}$ is increased by 8%, however, for f_{D_s} it is increased by up to 19%. So the ratio $f_{D_s^*}/f_{D_s}$ is decreased by 9%. In table 4 we summarized the values of decay constant $f_{D_s^*}$, f_{D_s} and the ratio of $f_{D_s^*}/f_{D_s}$ from various sources. Our result for f_{D_s} agrees well with other theoretical calculations and experimental results within errors. For the ratio $f_{D_s^*}/f_{D_s}$, our results have a value greater than 1, however, Browder et al. [17] has a value less than 1. It seems that this ratio is more likely to be greater than 1 when we consider that the decay constant of ρ meson is 1.5 times greater than that of π meson. The difference of the results by Cheng and Yang [19] comes from the different method and using different Wilson coefficients. Their values come by comparing two non-leptonic decay modes, for instance $\mathcal{B}(B \to DD_s(D^{(*)}D_s^*))/\mathcal{B}(B \to D\pi)$.

5. Conclusion

By including the penguin contributions and the non-factorizable term contributions, we calculated the weak decay constants f_{D_s} and $f_{D_s^*}$ from $\bar{B}^0 \to D^+\ell^-\nu$ and $\bar{B}^0 \to D^+\bar{D}_s^{(*)}$. In our analysis, we consider the QCD-penguin and Electroweak-penguin contributions in hadronic two body decays within the NDR renormalization scheme at next-to-leading order calculation. We also considered the effect of two different q^2-dependence of the form factor for $F_0^{BD}(q^2)$. The value of f_{D_s} is changed by less than 10% for different form factors.

The penguin effects for $B \to DD_s$ decay is quite sizable, and we obtained $f_{D_s} = 233 \pm 49$ MeV for the monopole type of F_0^{BD}, $f_{D_s} = 255 \pm 54$ MeV for the constant F_0^{BD}. When we considered the non-factorizable contributions, we obtained $f_{D_s^*} = 346 \pm 82$ MeV for the D_s^* meson. These values will be improved vastly when the large accumulated data samples are available at the Belle and BaBar experiments in near future.

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	$\mathcal{B}(\bar{B}^0 \to D^+\rho^-)$ $\times 10^3$	$\mathcal{B}(\bar{B}^0 \to D^+K^{-*})$ $\times 10^4$	$\mathcal{B}(\bar{B}^0 \to D^+\pi^-)$ $\times 10^3$	$\mathcal{B}(\bar{B}^0 \to D^+K^-)$ $\times 10^4$
(pole/pole)	9.01 ± 1.54	4.62 ± 0.79	3.58 ± 0.61	2.74 ± 0.47
(pole/const.)	9.01 ± 1.54	4.62 ± 0.79	3.57 ± 0.61	2.71 ± 0.46
Experiments	$8.4 \pm 1.6 \pm 0.7$	—	$3.1 \pm 0.4 \pm 0.2$	—

Table 1: The obtained values of the branching ratios with $a_1 = 1.02$ and experimental measurements.

Coefficients	Real Part	Imaginary Part
C_1^{eff}	1.168	0.0
C_2^{eff}	-0.365	0.0
C_3^{eff}	$2.25 \cdot 10^{-2}$	$4.5 \cdot 10^{-3}$
C_4^{eff}	$-4.58 \cdot 10^{-2}$	$-1.36 \cdot 10^{-2}$
C_5^{eff}	$1.33 \cdot 10^{-2}$	$4.5 \cdot 10^{-3}$
C_6^{eff}	$-4.80 \cdot 10^{-2}$	$-1.36 \cdot 10^{-2}$
C_7^{eff}	$2.37 \cdot 10^{-4}$	$-2.88 \cdot 10^{-4}$
C_8^{eff}	$4.30 \cdot 10^{-4}$	0.0
C_9^{eff}	$-1.11 \cdot 10^{-2}$	$-2.88 \cdot 10^{-4}$
C_{10}^{eff}	$3.75 \cdot 10^{-3}$	0.0

Table 2: The values of the effective wilson coefficient C_i^{eff} with the $\mu = m_b(m_b) = 4,3$ GeV, $m_c(m_b) = 0.95$ GeV in the NDR scheme at NLO calculation.

	$(N_c)_{LL}=2,\ \ (N_c)_{LR}=2$		$(N_c)_{LL}=2,\ \ (N_c)_{LR}=5$		$(N_c)_{LL}=3,\ \ (N_c)_{LR}=3$	
Coeffs.	Real Part	Imag. Part	Real Part	Imag. Part	Real Part	Imag. Part
a_1	0.985	0.0	0.985	0.0	1.046	0.0
a_2	0.219	0.0	0.219	0.0	0.024	0.0
a_3	-4.00$\cdot 10^{-4}$	-2.30 $\cdot 10^{-3}$	-4.00$\cdot 10^{-4}$	-2.30 $\cdot 10^{-3}$	7.23$\cdot 10^{-3}$	-3.30 $\cdot 10^{-5}$
a_4	-3.46 $\cdot 10^{-2}$	-1.14$\cdot 10^{-2}$	-3.46 $\cdot 10^{-2}$	-1.14$\cdot 10^{-2}$	-3.83 $\cdot 10^{-2}$	-1.12$\cdot 10^{-2}$
a_5	-1.07 $\cdot 10^{-2}$	2.3 $\cdot 10^{-3}$	3.70 $\cdot 10^{-3}$	1.78 $\cdot 10^{-3}$	-2.70 $\cdot 10^{-3}$	-3.33 $\cdot 10^{-5}$
a_6	-4.13 $\cdot 10^{-2}$	-1.14 $\cdot 10^{-2}$	-4.53 $\cdot 10^{-2}$	-1.27 $\cdot 10^{-2}$	-4.36 $\cdot 10^{-2}$	-1.21 $\cdot 10^{-2}$
a_7	-2.19 $\cdot 10^{-5}$	-2.88 $\cdot 10^{-4}$	-1.51 $\cdot 10^{-4}$	-2.88 $\cdot 10^{-4}$	-9.35 $\cdot 10^{-5}$	-2.88 $\cdot 10^{-4}$
a_8	3.11 $\cdot 10^{-4}$	-1.44 $\cdot 10^{-4}$	-3.82 $\cdot 10^{-4}$	-5.77 $\cdot 10^{-5}$	3.51 $\cdot 10^{-4}$	-9.61 $\cdot 10^{-5}$
a_9	-9.27 $\cdot 10^{-3}$	-2.88 $\cdot 10^{-4}$	-9.27 $\cdot 10^{-3}$	-2.88 $\cdot 10^{-4}$	-9.90 $\cdot 10^{-3}$	-2.88 $\cdot 10^{-4}$
a_{10}	-1.82 $\cdot 10^{-4}$	-1.44 $\cdot 10^{-4}$	-1.82 $\cdot 10^{-4}$	-1.44 $\cdot 10^{-4}$	3.39 $\cdot 10^{-5}$	-9.61 $\cdot 10^{-5}$

Table 3: The values of the effective coefficients a_i with $\mu = m_b(m_b) = 4,3$ GeV and $m_c(m_b) = 0.95$ GeV in the NDR scheme at NLO calculation. a_{2i} and a_{2i-1} are defined by $a_{2i-1} = C^{eff}_{2i-1} + C^{eff}_{2i}/N^{eff}_c$ and $a_{2i} = C^{eff}_{2i} + C^{eff}_{2i-1}/N^{eff}_c$. Here we have taken $(N_c)_{LL}$ for (V-A)(V-A) interaction and $(N_c)_{LR}$ for (V-A)(V+A) interaction.

	$(N_c)_{LL}$	$(N_c)_{LR}$	$f_{D_s^*}$ (MeV)	f_{D_s} (MeV)	$f_{D_s^*}/f_{D_s}$
(pole/pole)	2	2	346 ± 82	231 ± 48	1.57 ± 0.50
	2	5	346 ± 82	233 ± 49	1.56 ± 0.49
	3	3	325 ± 77	216 ± 45	1.57 ± 0.50
(pole/const.)	2	2	346 ± 82	252 ± 53	1.44 ± 0.45
	2	5	346 ± 82	255 ± 54	1.43 ± 0.45
	3	3	325 ± 77	235 ± 50	1.44 ± 0.45
Browder *et al.*[17]			243 ± 70	277 ± 77	0.88 ± 0.35
Hwang and Kim [18]			362 ± 15	309 ± 15	1.17 ± 0.02
Cheng and Yang [19]			266 ± 62	261 ± 46	1.02 ± 0.30
Capstick and Godfrey [20]				290 ± 20	
Dominguez [21]				222 ± 48	
UKQCD [22]				212^{+4+46}_{-3-7}	
BLS [23]				$230 \pm 7 \pm 35$	
MILC [24]				$199 \pm 8^{+40+10}_{-11-0}$	
Becirevic *et al.* [25]			$272 \pm 16^{+0}_{-20}$	$231 \pm 12^{+6}_{-0}$	1.18 ± 0.18
WA75 [26]				$238 \pm 47 \pm 21 \pm 48$	
CLEO 1 [27]				$282 \pm 30 \pm 43 \pm 34$	
CLEO 2 [28]				$280 \pm 19 \pm 28 \pm 34$	
BES [29]				$430^{+150}_{-130} \pm 40$	
E653 [30]				$190 \pm 34 \pm 20 \pm 26$	

Table 4: The obtained values of $f_{D_s^*}$ (MeV) and f_{D_s} (MeV), and their ratio $f_{D_s^*}/f_{D_s}$, and the results from other theoretical calculations and existing experimental results. Here we refered the corrected f_{D_s} values [31] for the experimental data [26] - [30].